etric function can now be expressed as:

$$l + 2,\ 2rn^{-1}) = \sum_{\nu=0}^{\infty} n^{-\nu}\, \Phi_{l,\nu}(r), \qquad (29)$$

$$+ 1)!\ \sum_{k=\nu}^{\infty} \frac{(-1)^k\, a_\nu^{k,l}\, (2r)^k}{k!\,(2l+1+k)!}. \qquad (30)$$

e transformed to sums of Bessel func-
s $a_\nu^{k,l}$ are written in a convenient form. It
t functions $\Phi_{l,\nu}$ are, when the abbreviation

$$(2l+1)!\ (\tfrac{1}{2}z)^{-2l-1} J_{2l+1}(z), \qquad (31)$$

$$\tfrac{1}{2}(2l+1)!\ (\tfrac{1}{2}z)^{-2l+1} J_{2l+1}(z), \qquad (32)$$

$$^{-2l-1}(8l^3+12l^2+4l)+(\tfrac{1}{2}z)^{-2l+1}(2l+2)+$$
$$-(\tfrac{1}{2}z)^{-2l}(4l^2+4l)-2(\tfrac{1}{2}z)^{-2l+2}\}\, J_{2l}(z)]. \qquad (33)$$

ons, $a_\nu^{k,l}/k!$ should be written down as a sum of
g. the form

$$\frac{{}^{1}\!/_{2}l+1}{2)!} + \frac{{}^{1}\!/_{2}l + {}^{5}\!/_{6}}{(k-3)!} + \frac{{}^{1}\!/_{8}}{(k-4)!} \qquad (34)$$

currence formulae for the Bessel func-

electronic levels, studied is this note, are the

$$(z), \qquad (35)$$

$$(36)$$

$$+ {}^{1}\!/_{8}({}^{1}\!/_{2}z)^3\}\, J_1(z) - {}^{1}\!/_{12}({}^{1}\!/_{2}z)^2\, J_0(z), \qquad (37)$$

$$J_3(z), \qquad (38)$$

$$J_3(z), \qquad (39)$$

$$+{}^{3}\!/_{4}({}^{1}\!/_{2}z)\}\, J_3(z)+\{-2({}^{1}\!/_{2}z)^{-2}-{}^{1}\!/_{2}\}\, J_2(z). \qquad (40)$$

the (E, r_0)-curve in the neighbourhood of
sary to consider the nodes r_0 of F or, by way
ain number of terms of the development (29)

$$^{-1}\,\Phi_{l,1} + n^{-2}\,\Phi_{l,2} = 0, \qquad (41)$$

$$= r_{00} + r_{01} + r_{02}, \qquad (42)$$

nd r_{01} and r_{02} of first and second order in n^{-1},
he function Φ in Taylor series and equa-

$$\Phi_{l,0}(r_{00}) = 0 \quad \text{or} \quad J_{l+1}(2\sqrt{2r_{00}}) = 0 \text{ gives } r_{00}, \qquad (43)$$

$$r_{01} = 0, \qquad (44)$$

$$\Phi'_{l,0}(r_{00})\, r_{02} + n^{-2}\,\Phi_{l,2}(r_{00}) = 0 \quad \text{gives } r_{02}. \qquad (45)$$

It follows by using some relations between Bessel functions and their derivatives, that for s-levels ($l = 0$):

$$r_{02} = \tfrac{1}{6} n^{-2}\, r_{00}^2, \qquad (46)$$

so that, in total:

$$r_0 = r_{00} + \tfrac{1}{6} n^{-2}\, r_{00}^2 = r_{00} - \tfrac{1}{3} E\, r_{00}^2; \qquad (47)$$

this being the equation of the tangent in $E = 0$ at the (E, r_0) curve for a s-level, with r_{00} following from the nodes of the Bessel function J_1.

The first node gives the tangent of the 1s-level:

$$r_0 = 1.835 - 1.123\, E, \qquad (48)$$

whereas the second node gives the tangent of the 2s-level:

$$r_0 = 6.153 - 12.620\, E. \qquad (49)$$

For the p-levels ($l = 1$) it is found after a simple calculus that

$$r_{02} = n^{-2}\,(\tfrac{1}{3}r_{00} + \tfrac{1}{6}r_{00}^2), \qquad (50)$$

and so for the tangent

$$r_0 = r_{00} + n^{-2}\,(\tfrac{1}{3}r_{00} + \tfrac{1}{6}r_{00}^2), \qquad (51)$$

$$r_0 = r_{00} - E(\tfrac{2}{3}r_{00} + \tfrac{1}{3}r_{00}^2), \qquad (52)$$

with r_{00} from $J_3(2\sqrt{2r_{00}}) = 0$.

For the 2p-level we need the first node of J_3, so that the tangent is

$$r_0 = 5.086 - 12.015\, E. \qquad (53)$$

The tangents (49) and (53) are indicated in figure 2.

d) $E > 0$. In the region of positive energies *), the confluent hypergeometric function (6) has imaginary parameters n and ρ (v.(2)). No tables for this region being available for $l = 0$ and $l = 1$, zero points have been calculated by using for the confluent hypergeometric function the series expansion of Buchholz [4]). The results are given in tables II–IV and plotted in figure 3.

e) $E \to \infty$. For the asymptotic case of small radii r_0 and thus large positive energies in the problem of the encaged hydrogen atom the influence of the proton on the electron can be neglected in

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*) The curve of reference 2 is only roughly sketched in that region and numerically not reliable.